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Critical Vacuum Energy, Warped Geometry and Grand Unification

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Abstract

We explore a mechanism to obtain the observational small value for the 4-dimensional vacuum energy through an exponential warp-factor suppression. Intriguingly the required suppression scale relates directly to the GUT scale. We demonstrate the mechanism explicitly in a 5-dimensional brane-world setup with warped geometry. Upon lifting the setup to 10-dimensional IIB string-theory, the relevance of the GUT scale becomes clear as the IIB string-theory description, which is based on D3-brane stacks, gives rise to a spontaneously broken SU(5) supersymmetric GUT theory with low-energy MSSM spectrum and Higgs doublet-triplet splitting.

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1 Introduction

The enormous smallness of the 4-dimensional vacuum energy, constrained by cosmological and astronomical measurements to be [1]

$$|\Lambda_4| \lesssim 10^{-47}\mathrm{GeV}^4 = (1.8\,\mathrm{meV})^4 \;, \tag{1}$$

is still not understood in a satisfactory way from a theoretical point of view. The energy-regime of the upper bound of some meV is rather unnatural in particle physics and more characteristic of condensed matter phenomena. However, it has to be noticed that the upper bound on the electron neutrino mass can be as low as 1meV [2], which comes strikingly close to this value. If experiment will eventually show that both numbers are indeed so close, it would be an intriguing hint to some deeper relation between the Standard Model (SM) and gravity.

The hope that eventually a consistent theory of quantum gravity might be able to explain the vexing smallness of the vacuum energy resp. cosmological constant has not been fulfilled yet, as the leading candidate, M/string-theory, relies so heavily on exact supersymmetry. Since the tininess of the cosmological constant is measured at energies where Bose-Fermi degeneracy is seen to be violated, a supersymmetry-breaking mechanism would be needed which nonetheless should not give rise to a large Λ_4. An interesting M-theory inspired proposal has been made in [3]. The idea is that in three dimensions, supersymmetry enforces a zero cosmological constant but can exist without matching bosonic and fermionic degrees of freedom. If such a 3-dimensional theory contains a modulus similar to the dilaton of string-theory, one could expect that at strong coupling an additional fourth dimension will open up much like in M-theory. The hope would be that during the transition from weak to strong coupling the zero cosmological constant and Bose-Fermi non-degeneracy might be preserved. Another interesting aspect which arose in string-theory is that vacua with zero and negative cosmological constant can sometimes be connected via T-duality [4]. This suggests that vacua with negative cosmological constant might in fact be viewed as flat spacetime vacua. Again this connection has so far only been found in three dimensions. Finally, there might also be a radically different understanding of the vacuum energy if M-theory turns out to be a theory of only a finite [5], [6] but huge amount of discrete degrees of freedom as suggested for instance by microscopic entropy considerations.

Whereas in the very early universe a large positive cosmological constant is welcome during the phase of inflation, we face the problem to understand the smallness of the

cosmological constant in our low-energy world today. Therefore, we shall take the point of view in this paper, that there should also exist a rationale to understand the adjustment of the cosmological constant to tiny values not only by taking refuge to a quantum gravity description valid at Planck-energies but also by employing merely degrees of freedom which are available at low energies.

Furthermore, we shall adopt the view that our 4-dimensional world arises as a brane-world from stacks of branes, embedded in some higher-dimensional spacetime. Conceiving our world as being located on a type IIB string-theory D3-brane in a 10-dimensional ambient space allowed to attack such fundamental problems as gauge and gravitational coupling unification or the Standard Model hierarchy problem from completely different point of views (see [7] and references therein) than the traditional technicolor or low-energy supersymmetry approaches. In a T-dualized type I string scenario, where two to six internal compact dimensions orthogonal to the D-brane are chosen much larger than the remaining compact dimensions, one is able to lower the fundamental higher-dimensional Planck scale down to the TeV scale [8]. This necessitates the large internal dimensions to be as large as 1mm resp. 1 fermi for two resp. six large internal dimensions. Most pronounced in the case of two large dimensions, this leads to another hierarchy between the new fundamental TeV scale and the compactification scale $\mu \equiv \hbar c/1\mathrm{mm} \approx 10^{-4}\mathrm{eV}$. This drawback could be overcome by considering not a direct product structure for the background space-time but a warped metric instead. In particular, the warped metric of a slice of an AdS-space suspended between two branes offers a solution to the strong part of the hierarchy problem [9].

In [10] it has been shown how to stabilize the modulus, which describes the distance between the two branes, at a value of 10-50 Planck lengths. This is the value which is compatible with the mentioned solution of the hierarchy problem. It remains to relax the fine-tuning condition between the bulk cosmological constant and the brane-tensions. Attempts in this direction have been undertaken recently (see e.g. [11], [12], [13]). However, the solution to the hierarchy problem cannot be maintained in these approaches as the solutions exhibit metrics that show polynomial instead of exponential behaviour. The metrics vanish at two finite points in the extra dimension, thereby cutting off the infinite range through singularities. However, the resolution of these singularities remains obscure.

A general review of the cosmological constant problem can be found in [14]. See [15], [16] for more recent reviews on the topic. [17] provides a recent discussion of the

cosmological constant problem from the point of view of string-theory. Apparently, lately there has been a noticeable increase in the efforts to address the cosmological constant problem [18]-[35].

The outline of this paper is as follows. In the next section, we lay the framework for determining the effective 4-dimensional vacuum energy Λ_4 by reanalyzing the Randall-Sundrum (RS) setup [9]. In the following section, we start from the observation that to obtain the critical meV sized Λ_4 through an exponential suppression from naturally arising vacuum energies with Planck-scale values, one requires in the exponent a suppression length directly related to the Grand Unified Theory (GUT) scale. A geometric brane-world realization of such an exponential suppression mechanism for Λ_4 is then given in terms of two branes embedded in a 5-dimensional ambient spacetime. The following section analyzes the influence of 5-dimensional bulk scalars on Λ_4 by deriving their effective 4-dimensional potential. In section 5 we embed the 5-dimensional setup into 10-dimensional IIB string-theory using stacks of D3-branes. The following sections discuss the general features of the string-theory description. Section 6 investigates how gauge and Higgs fields emerge from open strings attached to the D3-brane stacks. A direct consequence of the string-theory description is a mass hierarchy between color triplets and weak doublets in the Higgs sector. Section 7 explains how heavy GUT fields together with light MSSM matter fields can arise from open strings when the compactification manifold is non-simply connected. Section 8 shows that the open string spectrum contains the complete spectrum of a supersymmetric SU(5) GUT theory with gauge group broken down to the MSSM's $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Finally section 9 addresses the issue of supersymmetry breaking. In two appendices, we deal with generalizations of the brane setup to branes with unequal tensions and analyze the influence of bulk scalars on Λ_4 for this unequal tension case.

2 The Effective Vacuum Energy

Let us start by analyzing in detail the contributions to the effective 4-dimensional vacuum energy resp. cosmological constant in the RS scenario. The RS-model [9] lives in five dimensions and has two 3-branes with 4-dimensional worldvolume located at the fixed points of an $\mathbf{S}^1/\mathbf{Z}_2$ orbifold along the fifth direction. In between the 3-branes there is a bulk 5-dimensional anti de Sitter (AdS) spacetime. The Planck brane, on which the 4-dimensional graviton gets localized, sits at the first fixed-point, $x^5 = 0$ of the $\mathbf{Z}_2$ action,

whereas our 4-dimensional visible world originates from the SM brane, placed at the second fixed-point $x^5 = \pi r$. It is the SM brane on which the hierarchy problem gets solved by means of the exponential warp-factor of the AdS bulk geometry. The dominant contribution to the action of the two branes comes from the brane tensions T_{Pl}, T_{SM} [36]. Hence, if one is interested in a situation where the branes are close to their ground states, it is reasonable to neglect gauge-field, fermion or scalar contributions and write for the RS-Lagrangean[2] [9]

$$S_{RS} = -\int d^4x \int_0^{\pi r} dx^5 \Big(\sqrt{-G}\left(M_5^3 R + \Lambda\right) \\ + \sqrt{-g_{Pl}^{(4)}}\, T_{Pl}\delta(x^5) + \sqrt{-g_{SM}^{(4)}}\, T_{SM}\delta(x^5 - \pi r)\Big) \,. \tag{2}$$

Here it is understood that we have to integrate the bulk action over the interval[3] $[-\epsilon, \pi r + \epsilon]$ with ϵ infinitesimal, rather than $[0, \pi r]$, to incorporate the full delta-function sources of the boundaries. The 4-dimensional metrics $g_{SM}^{(4)}, g_{Pl}^{(4)}$ are the respective pullbacks of the bulk metric to the two 3-brane world-volumes. Adopting the metric Ansatz

$$ds^2 = e^{-A(x^5)}\eta_{\mu\nu}dx^\mu dx^\nu + (dx^5)^2 \,, \tag{3}$$

the Einstein equations lead to

$$(A')^2 = -\frac{1}{3M_5^3}\Lambda \,, \qquad A'' = \frac{1}{3M_5^3}\left(T_{Pl}\delta(x^5) + T_{SM}\delta(x^5 - \pi r)\right) \,. \tag{4}$$

The solution to the first equation is given by

$$A(x^5) = \pm k x^5 \,, \quad k \equiv \sqrt{\frac{-\Lambda}{3M_5^3}} \,, \tag{5}$$

where the integration constant has been set to zero. To respect the orbifold's $\mathbf{Z}_2$ symmetry, which sends $x^5 \to -x^5$, we have to take

$$A(x^5) = \pm k|x^5| \,. \tag{6}$$

In the following, we will choose the plus-sign which allows for a solution of the hierarchy problem on the SM-brane. Noting that $|x^5|'' = 2\delta(x^5)$, we rewrite the solution in an expanded form as

$$A(x^5) = \frac{1}{2}k\left(|x^5| - |x^5 - \pi r|\right) + \frac{1}{2}k\pi r \,, \qquad 0 \le x^5 \le \pi r \tag{7}$$

[2]Subsequently, we will adopt the general relativity conventions of [37].

[3]This is analogous to the downstairs approach in heterotic M-theory [38]. In the alternative upstairs approach, one would integrate the Lagrangean density over the full circle instead but has to add a factor of 1/2 in front of the bulk action.

in order to satisfy the second equation of (4) with

$$T_{Pl} = -T_{SM} = 3M_5^3 k \; . \tag{8}$$

Let us now determine the resulting 4-dimensional effective action by integrating out the fifth dimension. We will first carry this out for the Einstein-Hilbert term of the bulk action. For this purpose, consider first the general D-dimensional case with warped metric

$$ds^2 = G_{MN}dx^M dx^N = g^{(D-1)}_{\mu\nu} dx^\mu dx^\nu + (dx^D)^2 = f(x^D) g_{\mu\nu}(x^\rho)\, dx^\mu dx^\nu + (dx^D)^2 \; , \tag{9}$$

where μ, ν run over $1, \ldots, D-1$ and M, N over $1, \ldots, D$. The D-dimensional curvature scalar can then be decomposed as follows into the $(D-1)$-dimensional curvature scalar plus additional terms depending exclusively on x^D (f' denotes the derivative df/dx^D)

$$R(G) = \frac{1}{f} R(g) + \frac{1}{4}(D-1)\left((D-2)\big[(\ln f)'\big]^2 + 2(\ln f)'' + 2\frac{f''}{f} \right) \; . \tag{10}$$

In addition, we have to take into account a factor $\sqrt{-G} = f^{(D-1)/2}\sqrt{-g}$ in the measure of the action integral.

Specializing now to the RS case with $D = 5$ we have the metric

$$ds^2 = G_{MN}dx^M dx^N = e^{-A(x^5)} g_{\mu\nu}(x^\rho) dx^\mu dx^\nu + (dx^5)^2 \; . \tag{11}$$

Using (10) with $f(x^5) = e^{-A(x^5)}$, we get

$$\begin{aligned} S_{\mathrm{EH}} &= -\int d^4x \int_0^{\pi r} dx^5 \sqrt{-G} M_5^3 R(G) \\ &= -\int d^4x \sqrt{-g} M_5^3 \int_0^{\pi r} dx^5 \left(e^{-A} R(g) + e^{-2A}\left[5(A')^2 - 4A''\right]\right) \; . \end{aligned} \tag{12}$$

Since we will make use of this formula later on, we note that up to this point it is valid for any metric which is of the form (11). Choosing the RS-metric we obtain

$$\begin{aligned} S_{\mathrm{EH}} = -\int d^4x \sqrt{-g} M_5^3 \Bigg(& R(g) \int_0^{\pi r} dx^5 e^{-kx^5} \\ & + \int_0^{\pi r} dx^5 e^{-2kx^5} \left[5k^2 - 4k\left(\delta(x^5) - \delta(x^5 - \pi r)\right)\right] \Bigg) \; . \end{aligned} \tag{13}$$

Concerning the delta-function integration we perform the integration over the interval $[-\epsilon, \pi r + \epsilon]$ with ϵ infinitesimal. Thus the Einstein-Hilbert action contributes the terms

$$S_{\mathrm{EH}} = -\int d^4x \sqrt{-g} \left(M_4^2 R(g) - \frac{3}{2} M_5^3 k \left(1 - e^{-2k\pi r}\right) \right) \; , \tag{14}$$

to the 4-dimensional effective action, where $M_4^2 = 2M_5^3(1-e^{-k\pi r})/k$ denotes the 4-dimensional Planck-scale squared.

The second part of the reduction comprises the brane sources and the bulk cosmological constant term

$$
\begin{aligned}
S_{Pl} + S_{SM} + S_\Lambda &= -\int d^4x\sqrt{-g}\left(e^{-2k\pi r}T_{SM} + T_{Pl} + \Lambda\int_0^{\pi r} dx^5 e^{-2kx^5}\right) \\
&= -\frac{3}{2}\int d^4x\sqrt{-g}M_5^3 k\left(1-e^{-2k\pi r}\right) \ ,
\end{aligned}
$$

where we have used (8) in the last row. To obtain the effective potential in the RS-scenario we add both contributions. Because $R(g)$ vanishes and due to the fine-tuning of the Planck and SM brane tensions in terms of the bulk cosmological constant (8), both contributions add up to zero and we obtain a zero Λ_4, as expected.

There are two interesting observations at this point. First, the above calculation shows that if one relaxes the fine-tuning (8) of the brane tensions but still assumes that they are equal in magnitude and of opposite sign, then one expects a residual 4-dimensional vacuum energy of order

$$\Lambda_4 \approx \pm M_5^3 k(1-e^{-2k\pi r}) \ . \tag{15}$$

where the sign depends on whether the bulk cosmological constant Λ is larger than the brane tensions (minus sign) or smaller (plus sign). Such an effective Λ_4 would constitute a potential for the interval length modulus r. For the plus sign choice its minimum lies at $r=0$ and would drive Λ_4 to zero[4] (for the minus sign choice the minimum would lie at $r=\infty$ implying a runaway behavior). However, an estimate of how close r has to come to zero to reconcile the vacuum energy with its observable value is rather disenchanting. If we take $M_5^3 k \approx M_{Pl}^4$, $k \approx M_{Pl}$ and demand that $\Lambda_4 \approx 1\text{meV}^4$, we find an incredibly small $r \approx 10^{-125} l_{Pl}$, where $l_{Pl} = M_{Pl}^{-1}$ with the Planck-mass given by $M_{Pl} = 1.2\times 10^{19}\,\text{GeV}$. This, however, is a region, where we surely cannot trust classical gravity any longer as a reliable framework.

Second, one observes that the warp-factor enters M_4^2 and Λ_4 differently. This means that the exponential warp-factor contribution to the cosmological constant $\lambda_4 = \Lambda_4/M_4^2$ does not drop out and therefore presents an interesting possibility to influence the effective 4-dimensional cosmological constant if one could get rid of the constant r-independent terms which exceed the exponential terms. In the rest of this paper we describe a 5-

[4]Note that in this limit the hierarchy-problem couldn't be solved any longer.

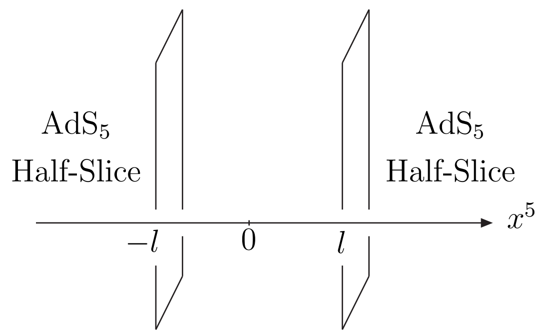

Figure 1: The double 3-brane setup with interbrane separation $2l$. To obtain a critical vacuum energy one has to set $2l \approx 1/M_{\mathrm{GUT}}$.

dimensional brane-world scenario where we study this warp-factor influence on λ_4 or equivalently the vacuum energy Λ_4 and its implications for string- and particle-theory.

3 The 5-Dimensional Two Brane Model

In the previous section we saw that the asymmetric (with respect to the $\mathbf{Z}_2$ orbifold symmetry acting on the fifth dimension) positioning of the Planck brane at $x^5 = 0$ and the SM brane at $x^5 = \pi r$ led, together with the choice of asymmetric tensions for these two branes, to an asymmetric warp-factor. It is this asymmetry which generated the unwanted constant term (from the Planck brane) next to the r-dependent wanted exponential term (from the SM brane) in the 4-dimensional vacuum energy (15). To obtain a small Λ_4 one should hence avoid placing a brane at the origin, the fixed point of the $\mathbf{Z}_2$ symmetry. Instead, we will place the two branes at the $\mathbf{Z}_2$ mirror-symmetric points $x^5 = -l$ and $x^5 = l$. We will see that in this way one can achieve an exponential suppression of Λ_4 at the expense of one classical fine-tuning. In contrast to the RS-model we will take x^5 to be non-compact, much as in the second model of Randall and Sundrum [39]. To respect the $\mathbf{Z}_2$ symmetry both branes will be given the same positive tension T (see fig. 1). Since we are assuming a non-compact x^5 coordinate, it is consistent to have both tensions positive[5]. In the bulk we will adopt a piecewise constant cosmological constant $\Lambda(x^5)$ so that the complete 5-dimensional action becomes

[5]Only for closed, i.e. compact and without boundary, dimension x^5 can one show that the sum of all brane tensions has to vanish [40].

$$
\begin{aligned}
S = &- \int d^4x \int dx^5 \sqrt{-G} \left(M_5^3 R(G) + \Lambda \right) \\
&- \int d^4x \int dx^5 \left(\sqrt{-g_1^{(4)}} T_1 \delta\left(x^5 + l\right) + \sqrt{-g_2^{(4)}} T_2 \delta\left(x^5 - l\right) \right) \ . \qquad (16)
\end{aligned}
$$

where we take $T_1 = T_2 = T$ and relegate the case with unequal tensions to appendix A. Again $g^{(4)}_{1,\mu\nu}$ and $g^{(4)}_{2,\mu\nu}$ are the induced metrics arising from the pullback of the bulk metric G_{MN} to the two brane world-volumes.

Choosing once more the Ansatz

$$
ds^2 = e^{-A(x^5)} \eta_{\mu\nu} dx^\mu dx^\nu + (dx^5)^2 \ , \qquad (17)
$$

the Einstein field equations reduce to the two equations (4) which read in our case

$$
(A')^2 = -\frac{1}{3M_5^3} \Lambda(x^5) \ , \qquad A'' = \frac{1}{3M_5^3} \left(T\delta\left(x^5 + l\right) + T\delta\left(x^5 - l\right) \right) \ . \qquad (18)
$$

The solution to these equations is given by

$$
A(x^5) \ = \ \frac{k}{2}\left|x^5 + l\right| + \frac{k}{2}\left|x^5 - l\right| \ = \ \begin{cases} -kx^5 & , \ x^5 \leq -l \\ kl & , \ -l \leq x^5 \leq l \\ kx^5 & , \ x^5 \geq l \end{cases} \ , \qquad (19)
$$

together with a bulk cosmological constant

$$
\Lambda(x^5) \ = \ \begin{cases} \Lambda & , \ |x^5| > l \\ \Lambda/4 & , \ |x^5| = l \\ 0 & , \ |x^5| < l \end{cases} \ = \ \begin{cases} -3M_5^3 k^2 & , \ |x^5| > l \\ -3M_5^3 k^2/4 & , \ |x^5| = l \\ 0 & , \ |x^5| < l \end{cases} \qquad (20)
$$

and brane-tension

$$
T = 3M_5^3 k \ . \qquad (21)
$$

Here we have set the integration constant, which could have been added to $A(x^5)$, to zero which amounts to one fine-tuning at the classical level[6]. The relation between the bulk cosmological constant in the exterior AdS_5 half-patches and the 3-brane tension becomes

$$
\Lambda = -\frac{1}{3} \frac{T^2}{M_5^3} \ . \qquad (22)
$$

[6]The same solution but with another choice for the undetermined integration constant has been obtained in [41]. The analysis in this work focussed on the localization of the graviton and corrections to the Newtonian gravitational potential.

The $\mathbf{Z}_2$ symmetric function $A(x^5)$ which determines the warp-factor is displayed in fig.2. Notice that the warp-factor is smaller than e^{-kl} everywhere

$$e^{-A(x^5)} \leq e^{-kl} \; . \tag{23}$$

For a low-energy observer at energies below $1/2l$, which we will soon identify with the

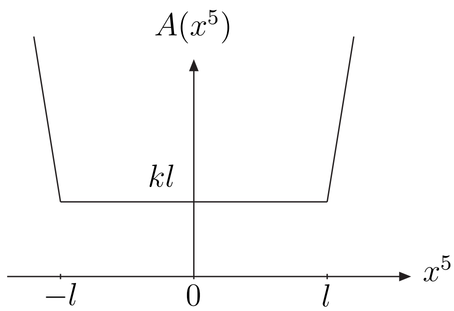

Figure 2: The function $A(x^5)$ which determines the warp-factor along the non-compact fifth dimension.

GUT scale, the separation between the two 3-branes becomes invisible. As a result he will see a geometry of two slices of AdS_5 spacetime glued together. For him the graviton would hence appear localized on the merged two 3-branes as described in [39].

Next, let us derive the effective 4-dimensional action by integrating over the x^5 coordinate in the action. For this we adopt the slightly more general background

$$ds^2 = e^{-A(x^5)} g_{\mu\nu}(x^\rho) dx^\mu dx^\nu + (dx^5)^2 \; . \tag{24}$$

where the flat $\eta_{\mu\nu}$ is replaced by a general $g_{\mu\nu}$, thus also allowing for 4-dimensional spacetimes with non-vanishing Λ_4. Along the same lines as for the RS-case we obtain by using (12) for the Einstein-Hilbert term

$$\begin{aligned} S_{EH} &= -\int d^4x \sqrt{g} M_5^3 \left(R(g) \int_{-\infty}^{\infty} dx^5 e^{-A} + \int_{-\infty}^{\infty} dx^5 e^{-2A} \left[5(A')^2 - 4A'' \right] \right) \\ &= -e^{-kl} \int d^4x \sqrt{g} M_5^3 \left(2R(g) \left[\frac{1}{k} + l \right] - 3e^{-kl} k \right) \; . \end{aligned} \tag{25}$$

Combining the two brane actions and the bulk cosmological constant gives furthermore

$$S_{SM_1} + S_{SM_2} + S_\Lambda = -e^{-2kl} \int d^4x \sqrt{g} \left(2T + \frac{\Lambda}{k} \right) \; . \tag{26}$$

Taken together the total 4-dimensional effective action becomes

$$
\begin{aligned}
& S_{EH} + S_{SM_1} + S_{SM_2} + S_\Lambda \\
= & -e^{-kl} \int d^4x \sqrt{g} \left(2M_5^3 R(g) \left[\frac{1}{k} + l \right] + e^{-kl} \left[-3M_5^3 k + 2T + \frac{\Lambda}{k} \right] \right) . \qquad (27)
\end{aligned}
$$

We will now drop the overall constant scale-factor e^{-kl} since it drops out of the equations of motion. One can also show by replacing $(dx^5)^2$ in the Ansatz for the metric by the more general $e^{-B(x^5)}(dx^5)^2$ that this constant overall factor can be absorbed without loss of generality into the definition of x^5. Our final 4-dimensional effective action therefore reads

$$
S_{D=4} = - \int d^4x \sqrt{g} \left(M_4^2 R(g) + \Lambda_4 \right) , \qquad (28)
$$

with the effective Planck-scale M_4 and vacuum energy Λ_4 given by

$$
M_4^2 = 2M_5^3 \left(\frac{1}{k} + l \right) \qquad (29)
$$

$$
\Lambda_4 = e^{-kl} \left(-3M_5^3 k + 2T + \frac{\Lambda}{k} \right) . \qquad (30)
$$

The remaining exponential factor e^{-kl} which occurs only in the vacuum energy will play an important role soon. But before coming to that, let us quickly verify our result by plugging in the values (20), (21) for T, Λ of our solution (19) which guaranteed a flat 4-dimensional Minkowski background. Therefore, thanks to the tuning of these parameters expressed by the relations (21) and (22), the 4-dimensional vacuum energy Λ_4 must vanish. This is indeed what we find with the above expression for Λ_4 and serves as a check on its derivation.

The important point is however the following. Suppose we lift the finetuning imposed on the parameters Λ and T. The 4-dimensional metric would then be no longer flat and the background becomes[7]

$$
ds^2 = e^{-A(x^5)} g_{\mu\nu} dx^\mu dx^\nu + (dx^5)^2 . \qquad (31)
$$

Without tuning, the square bracket in (30) will generically assumes positive or negative values of order M_{Pl}^4. In this paper we want to focus on the positive values. Taking the fundamental 5-dimensional Planck scale at $M_5 = M_{Pl}$, it will be natural to have also $k = M_{Pl}$. Note that the bulk cosmological constant will stay zero in between the 3-branes

[7] It has been shown in [28] that the full backreaction of the non-finetuned parameters preserves this warp-factor structure of the metric.

if we leave the $\mathbf{Z}_2$ symmetry of the setup intact [28]. The 4-dimensional vacuum energy for non-tuned parameters will then be non-zero and of the form

$$\Lambda_4 = e^{-kl} M_{Pl}^4 \, . \tag{32}$$

This is an interesting result since the exponential factor allows to lower the enormous Planck sized vacuum energy down to small values in a natural way, i.e. without invoking a new large hierarchy. So what is the appropriate distance $2l$ between the two 3-branes which allows to bring a Planck sized vacuum energy down to the critical meV scale? With $k = M_{Pl}$ this length coincides quite precisely with the inverse GUT unification scale

$$\boxed{\Lambda_4 \approx \text{meV}^4 \qquad \Leftrightarrow \qquad 2l \approx M_{\text{GUT}}^{-1}} \tag{33}$$

which is given by $M_{\text{GUT}} = 2 \times 10^{16}\,\text{GeV}$ or $M_{\text{GUT}} = (568 l_{Pl})^{-1}$ in terms of the Planck length[8]. It is intriguing that the required length turns out to be so natural which strongly suggests some GUT theory connection. We will make this connection explicit later when we embed the 5-dimensional setup into 10-dimensional IIB string-theory.

Let us take stock of what has been achieved so far. Using one classical tuning to set the integration constant in (19) to zero (one might hope to find a dynamical reason for this natural choice), we obtain with the warped geometry a mechanism to exponentially suppress the generically Planck sized vacuum energy down to critical meV^4 size. In particular this allows to suppress quantum corrections to the 4-dimensional vacuum energy coming from fields on the 3-branes, which renormalize the brane's tension T, without the need to readjust the resulting vacuum energy order by order in perturbation theory. Besides suppressing the contributions to the 4-dimensional vacuum energy coming from classical bulk contributions and classical plus quantum contributions of gauge and matter fields located on the 3-branes, it would be very interesting to investigate whether the suppression mechanism also extends to quantum contributions coming from bulk fields. We will leave this interesting aspect to future research but verify as a first step in this direction in the next section that generic bulk fields do not spoil the suppression mechanism at the classical level. The intriguing outcome of this section is furthermore that this warped geometry vacuum energy suppression mechanism points directly towards a connection between the meV^4 critical vacuum energy Λ_4, the Planck scale M_{Pl} and the GUT unification scale M_{GUT}. The role of GUT theories will be explored in sections 5 to 8.

[8] Notice that because $2l \gg l_{Pl}$ we can trust the field theory framework.

4 The Effective Potential from Bulk Scalars

An embedding of the 5-dimensional setup into IIB string-theory or F-theory along the lines of [42] requires in general additional bulk scalar fields in the 5-dimensional theory coming from the decomposition of 10-dimensional fields upon dimensional reduction from ten to five dimensions. It is important to check that such additional bulk fields do not reintroduce unsuppressed Planck-scale contributions to the 4-dimensional vacuum energy. Otherwise the warp-factor suppression mechanism of the 4-dimensional vacuum energy described so far could only be realized in simple 5-dimensional brane world scenarios without bulk scalars but not be embedded into string-theory. We will show in this section that generic bulk scalars do not spoil the suppression mechanism.

For this let us now examine the 4-dimensional effective potential, in the same warped background as before, generated by a canonical 5-dimensional bulk scalar Φ with quartic couplings to the two 3-branes. Such couplings are for instance required by the Goldberger-Wise mechanism [10] to stabilize the fifth dimension. The action for this scalar reads

$$\begin{aligned} S_\Phi = &- \int d^4x \int_{-\infty}^{\infty} dx^5 \sqrt{G} \Big(\frac{1}{2} G^{MN} \partial_M \Phi \partial_N \Phi + \frac{1}{2} m^2 \Phi^2 \Big) \\ &- \int d^4x \int_{-\infty}^{\infty} dx^5 \Big(\sqrt{g_1^{(4)}} \lambda_1 (\Phi^2 - v_1^2)^2 \delta(x^5 + l) + \sqrt{g_2^{(4)}} \lambda_2 (\Phi^2 - v_2^2)^2 \delta(x^5 - l) \Big) , \end{aligned}$$

with m the scalar's mass and positive couplings λ_1, λ_2. We will assume that Φ depends only on x^5 and study it in the fixed gravitational background given by our solution (17), (19). In this background we arrive at the following equation of motion for Φ

$$\begin{aligned} (e^{-2A(x^5)} \Phi')' - e^{-2A(x^5)} m^2 \Phi = 4 \big[& e^{-2A(-l)} \lambda_1 (\Phi^2 - v_1^2) \Phi \delta(x^5 + l) \\ & + e^{-2A(l)} \lambda_2 (\Phi^2 - v_2^2) \Phi \delta(x^5 - l) \big] , \end{aligned} \tag{34}$$

which has the solution

$$\Phi(x^5) = \begin{cases} a e^{(1+\Gamma)A(x^5)} + b e^{(1-\Gamma)A(x^5)} & , \ x^5 < -l \\ c e^{m x^5} + d e^{-m x^5} & , \ |x^5| \le l \\ e e^{(1+\Gamma)A(x^5)} + f e^{(1-\Gamma)A(x^5)} & , \ x^5 > l \end{cases} , \tag{35}$$

with

$$\Gamma = \sqrt{1 + m^2/k^2} \tag{36}$$

and free coefficients a, b, c, d, e, f.

In order to obtain a normalizable solution we are forced to set $a = e = 0$. Furthermore, demanding continuity of Φ at the location of the 3-branes determines b and f in terms of c and d

$$b = e^{(\Gamma-1)kl}\tilde{b} \ , \quad \tilde{b} = ce^{-ml} + de^{ml} \tag{37}$$

$$f = e^{(\Gamma-1)kl}\tilde{f} \ , \quad \tilde{f} = ce^{ml} + de^{-ml} \ . \tag{38}$$

To fix the remaining free coefficients c and d one could plug the above solution into the field equation and integrate it over the fifth dimension to incorporate brane boundary conditions. However, this leads to a complicated cubic equation in the unknowns c and d. An easier way to arrive at a determination of the coefficients c and d, proposed by [10], is to insert the scalar field solution into the scalar's action and integrate over x^5 to arrive at an effective potential for the interbrane distance $2l$. The minimization of this effective potential will then determine c and d. We will now follow this strategy. From the couplings of Φ to the 3-branes the effective potential receives the contributions

$$\int d^4x \left(\sqrt{g_1^{(4)}}\lambda_1 \left(\Phi^2(-l) - v_1^2\right)^2 + \sqrt{g_2^{(4)}}\lambda_2 \left(\Phi^2(l) - v_2^2\right)^2 \right) \ . \tag{39}$$

To minimize this potential for positive couplings λ_1, λ_2 we must set $\Phi(-l) = v_1$ and $\Phi(l) = v_2$. These two conditions finally determine c and d as

$$c = \frac{-v_1 e^{-ml} + v_2 e^{ml}}{2\sinh(2ml)} \ , \qquad d = \frac{v_1 e^{ml} - v_2 e^{-ml}}{2\sinh(2ml)} \ . \tag{40}$$

With all coefficients in the solution for Φ being fixed, the effective 4-dimensional potential V_Φ, defined by $S_\Phi = -\int d^4x\sqrt{g}V_\Phi(l)$, becomes

$$V_\Phi(l) = \frac{e^{-2kl}}{2}\left((v_1^2 + v_2^2)\left[(\Gamma-1)k + m\coth(2ml)\right] - 2v_1v_2\frac{m}{\sinh(2ml)} \right) \ , \tag{41}$$

where the identity

$$(1-\Gamma)^2k^2 + m^2 = 2\Gamma(\Gamma-1)k^2 \tag{42}$$

has been utilized. Usually when performing a dimensional reduction of a string-theory model, we retain only the massless modes with $m = 0$ in the low-energy effective action. For these the effective potential generated by Φ simplifies to

$$V_\Phi(l) = \frac{e^{-2kl}}{4l}(v_1 - v_2)^2 \ . \tag{43}$$

Therefore both in the massive and massless case, the important exponential suppression-factor is present (again only one e^{-kl} remains after discarding an overall e^{-kl} factor of the

action as explained earlier). Hence, for the same distance between the 3-branes as before, $2l = M_{\rm GUT}^{-1}$, bulk scalars with values for v_1, v_2, m up to the Planck scale will not introduce contributions to the 4-dimensional vacuum energy larger than the critical one in virtue of (32), (33).

As an aside, let us ask whether the effective potential obtained from a bulk scalar may stabilize the interbrane distance $2l$. From (43) it is immediately recognizable that in the massless case no minimum at finite l exists. In the massive case, setting $\partial V_\Phi/\partial l$ equal to zero, leads to the following equation for l

$$(w^2+1)\left(\left[\frac{\Gamma-1}{r}+\cosh(2ml)\right]\sinh(2ml)+r\right)=2w\left(r\cosh(2ml)+\sinh(2ml)\right)\ , \quad (44)$$

where we have employed the dimensionless ratios

$$w=\frac{v_1}{v_2}\ , \qquad r=\frac{m}{k}\ , \qquad (45)$$

in terms of which we can write $\Gamma=\sqrt{1+r^2}$. A numerical analysis of this equation for generic values of v_1, v_2, m shows that there are no solutions for l which would be real and positive. We can therefore conclude that in the massive case the effective potential exhibits no minimum either. Thus bulk scalars cannot be used for a stabilization of l. The case with different 3-brane tensions $T_1 \neq T_2$ will be analyzed in appendix B. Let us note that for the IIB string-theory embeddings which we will discuss in the next section following the uplifting of RS-scenarios as described in [42], only scalars are generated in five dimensions upon dimensional reduction of the internal metric and other fields including the axio-dilaton, 3-form and 5-form fluxes.

The inability of the scalars to stabilize the fifth dimension open up, however, a potential relevance for cosmology. For $2l = M_{\rm GUT}^{-1}$ we find that bulk scalars induce an extremely tiny (since exponentially suppressed) but non-vanishing repulsive force between the 3-branes such that the setup might be regarded as quasi-static. On the other hand for much smaller lengths $2l \approx 0$ two things will happen. First, the repulsive force will be much larger, driving the two 3-branes apart very quickly, hence leading to a fast time-dependent cosmological evolution. Second, the vacuum energy will no longer be suppressed as the exponential factor becomes unity. This seems to fit well with expectations about the very early universe, where to start inflation a considerable nonvanishing 4-dimensional vacuum energy $\Lambda_4 = V(\phi)$ is needed, $V(\phi)$ being the potential or vacuum energy density of the inflaton ϕ. For example in the scenario of chaotic inflation [43] one indeed requires a Planck size potential $V(\phi) \approx M_{Pl}^4$ which could arise when $2l \approx 0$. And there is another

aspect which fits nicely in. Namely at $2l \approx 0$ both 3-branes lie essentially on top of each other. If we jump a bit ahead and identify the 3-branes with D3-branes in IIB string-theory, then we know that putting them on top of each other implies a gauge symmetry enhancement which, as will be discussed later, could describe a GUT unification. This then suggests the following cosmological scenario. In the very early universe when both 3-branes are close together we have an unbroken GUT unification group and a huge cosmological constant, potentially capable of driving inflation. Due to the large repulsive force between the 3-branes they initially separate rapidly along the fifth direction. However, the separation process slows down soon due to the exponential suppression of the repulsive interbrane force. Today these forces have become miniscule and the brane setup evolution quasi-static with interbrane distance $2l$ having reached M_{GUT}^{-1} giving a small critical vacuum energy. Moreover, the GUT unification group will be broken today upon identification of the 3-branes with D-branes. An evolution along these lines might also arise in heterotic M-theory where forces between its two boundaries depend similarly on their distance [44]. We will not investigate these cosmological aspects further in this work and will now discuss the string-theory embedding.

5 Lift to IIB String-Theory

We have seen the important role played by the GUT unification scale in the suppression of the vacuum energy to achieve the critical value. It arose geometrically as the inverse of the length between the two 3-branes and strongly suggests a GUT theory connection. This connection and the GUT theory will become transparent once the 5-dimensional setup is embedded into IIB string-theory resp. F-theory compactified on a Calabi-Yau three-fold resp. four-fold. The 3-branes become D3-branes and open strings stretching between them over a length of the inverse GUT scale give naturally rise to X, Y leptoquark gauge bosons with masses at or above M_{GUT}. In consistency with the fact that we are addressing the vacuum energy not at early epochs of the universe but today, we will find a GUT theory with broken symmetry and consequently heavy X and Y leptoquark gauge bosons. We will discuss the string-theory GUT connection and related issues in this and the remaining sections.

The low-energy 5-dimensional geometry consists of two half infinite AdS_5 patches with an interpolating flat spacetime interval. Since an AdS_5 geometry arises as the near-horizon geometry of a type IIB string-theory D3-brane, one is naturally led to consider

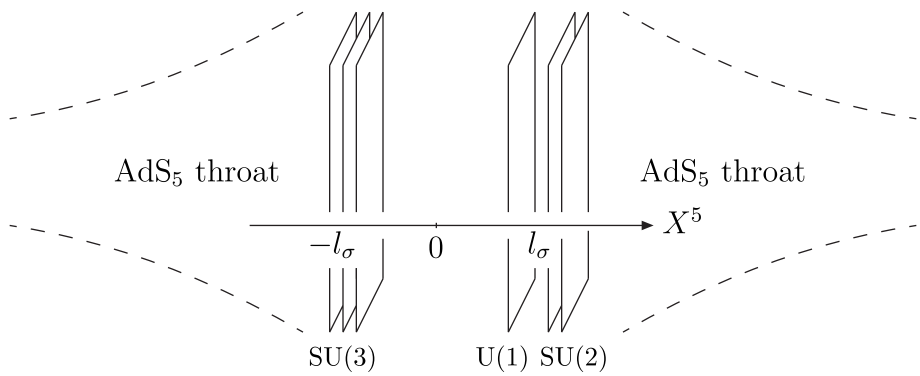

Figure 3: The two 3-branes resolved as stacks of D3-branes in the microscopic IIB string-theory picture.

IIB string-theory as the 10-dimensional parent theory and replace the 3-branes by D3-branes. Moreover, since we had two 3-branes of equal tension, we should consider two stacks of D3-branes with the same amount of D3-branes in either stack. Now we have to decide how many D3-branes should be in each stack. For this let us note that we also need to accomodate the SM with gauge group $[\mathrm{SU}(3)_c] \times [\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y]$. With two stacks of an equal number of D3-branes this can be achieved by having three D3-branes in each stack. The QCD color group will arise from one stack while the electroweak gauge group arises from the other. The split into the weak $\mathrm{SU}(2)_L$ and electromagnetic $\mathrm{U}(1)_Y$ gauge group requires a tiny further split between the 3 branes of the second stack into 2 giving rise to $\mathrm{SU}(2)_L$ and a single one responsible for $\mathrm{U}(1)_Y$ (see fig.3). By X^5 we will denote the string-theory target space coordinate which relates upon reduction to five dimensions to x^5. Each stack of D3-branes gives rise to a further local U(1) symmetry which is related to the center of mass position of the stack. For these two and the $\mathrm{U}(1)_Y$ gauge group it can be shown that only one can stay anomaly-free [45], [46]. $\mathrm{U}(1)_Y$ will later be identified with the anomaly-free hypercharge gauge group since the SM matter fields will turn to transform under it with the correct hypercharges. The anomalies of the two other abelian U(1) groups are cancelled in string-theory by the Green-Schwarz mechanism, which renders them massive. They remain as global symmetries with the mass of the corresponding U(1) gauge-bosons shifted to the string-scale.

This general recipe for lifting 5-dimensional AdS$_5$ geometries to 10-dimensional IIB string-theory setups containing D3-branes whose near-horizon geometries give rise to the

AdS_5 throats has been proposed in [42] and will be used here. Indeed for the $\mathbf{Z}_2$ symmetric D3-brane configuration depicted in fig.3, it has been argued in [42] that the D3-brane stacks, positioned at $X^5 = \pm l_\sigma$ in string-frame, each lead to a half-infinite AdS_5 patch in the effective 5-dimensional description. Even though one starts with a compactification on a compact 6-manifold K_6 for which the range of X^5 is compact, the throats are governed by warp-factors which can map a compact X^5 domain into a semi-infinite non-compact x^5 range. Note that the D3-branes are 4-dimensional spacetime filling and appear thus as points on the compactification manifold.[9]

Because the D3-branes appear as points on the internal 6-manifold K_6 (or elliptically fibered Calabi-Yau four-fold in the F-theory description) they are not sensitive to the global properties of the compactification manifold. It is only the number of D3-branes which has an influence on global properties via the tadpole cancellation condition [47],[48] which expresses the conservation of Ramond-Ramond (RR) 5-form flux. For F-theory compactifications on an elliptically fibered Calabi-Yau fourfold K_8 with K_6 as the base for the corresponding IIB compactification, the tadpole cancellation condition states that the Euler-characteristic χ of K_8, the background fluxes and the number of D3-branes N_{D3} have to satisfy

$$N_{D3} = \frac{\chi(K_8)}{24} - \int_{K_6} \frac{1}{2i\tau_2} H \wedge \bar{H} \ . \tag{46}$$

In our case $N_{D3} = 6$ and the 3-forms $H, \bar{H}$ are given as linear combinations of the RR and Neveu-Schwarz (NS) 3-form field-strengths

$$H = H^{RR} - \tau H^{NS} \ , \quad \bar{H} = H^{RR} - \bar{\tau} H^{NS} \ , \tag{47}$$

with $\tau = \tau_1 + i\tau_2 = a + ie^{-\phi}$ the axio-dilaton modulus of the elliptic fibration containing axion a and dilaton ϕ.

The open strings stretching between both D3-brane stacks will lead to massive states with mass

$$M_{\sigma,\mathrm{open}} = 2 l_\sigma T \tag{48}$$

when measured in string-frame. Here $T = (2\pi\alpha')^{-1}$ is the string-tension. We would like to know what the corresponding mass is when it is measured in the low-energy frame where an experimentalist would access it. For this we have to relate the IIB string-frame metric G^σ_{AB} ; $A, B = 1, \ldots, 10$ and the low-energy metric G_{AB} which is used to measure length in the 5-dimensional scenario. This is done via the Einstein-frame metric [42]. In

[9] For approaches to embed effective brane configurations into supergravity see [49], [50], [51], [52].

ten dimensions string-frame and Einstein-frame metric are related through

$$G^E_{AB} = e^{-\frac{\phi}{2}} G^\sigma_{AB} \, , \tag{49}$$

whereas the low-energy metric $(m, n = 6, \ldots, 10)$

$$ds^2 = G_{AB} dx^A dx^B = e^{-A(x^5)} g_{\mu\nu}(x^\rho) dx^\mu dx^\nu + (dx^5)^2 + h_{mn}(x^5, y^k) dy^m dy^n \tag{50}$$

is related to G^E_{AB} by a further Weyl-rescaling involving the internal 5-dimensional volume [42]

$$G_{AB} = V_5^{1/4} G^E_{AB} \, , \qquad V_5 = \frac{1}{L^5_{Pl}} \int_{K_5} d^5 y \sqrt{h} \, , \qquad h = \det h_{mn} \, . \tag{51}$$

Here $L_{Pl} = g_s^{1/4} \sqrt{\alpha'} (2\pi)^{7/8}$ denotes the 10-dimensional Planck-length with string coupling constant $g_s = e^\phi$ and Regge slope α'. K_5 stands for those 5-dimensional sections of the base-manifold K_6 for which x^5 is held constant. The effect of these rescalings is a simple expression for the 5-dimensional Planck-mass M_5 in terms of L_{Pl}, which can be read off from the Einstein-Hilbert action

$$-\frac{1}{(2\pi)^7 (\alpha')^4} \int d^{10} X \sqrt{-G^\sigma} R^\sigma = -\frac{1}{L^8_{Pl}} \int d^{10} x \sqrt{-G^E} R^E = -\frac{1}{L^8_{Pl}} \int d^{10} x \frac{\sqrt{-G} R}{V_5} \tag{52}$$

and upon dimensional reduction to five dimensions leads to the identification

$$M_5 = L^{-1}_{Pl} \, . \tag{53}$$

Substituting this result for M_5 into equ. (29) allows us to determine the value of the 10-dimensional Planck-length

$$L_{Pl} = \Big(\frac{2(1+kl)}{k M_4^2} \Big)^{1/3} \simeq \Big(\frac{2l}{M_4^2} \Big)^{1/3} = \frac{1}{(M_4^2 M_{\text{GUT}})^{1/3}} = \frac{1}{4 \times 10^{17} \text{GeV}} \, , \tag{54}$$

where we have used $2l = M^{-1}_{\text{GUT}}$ which implies $2kl = k/M_{\text{GUT}} \gg 1$ for generic values k around M_{Pl} and have identified M_4 with its actual value $M_4 = M_{Pl}/\sqrt{16\pi} = 1.7 \times 10^{18}$ GeV. Having determined the value for L_{Pl} (and at the same time for M_5), we obtain from its expression in terms of string-theory parameters the following relation between the string-scale $M_s = 1/\sqrt{\alpha'}$ and the string-coupling constant

$$M_s = 4 (2\pi)^{7/8} g_s^{1/4} \times 10^{17} \, \text{GeV} = 2 \times 10^{18} g_s^{1/4} \, \text{GeV} \, . \tag{55}$$

Moreover, we find from the relation between the string-frame and low-energy metric that the inter 3-brane distance $2l$ in the 5-dimensional description and the corresponding length $2l_\sigma$ in the string description are related by

$$2l = V_5^{1/8} e^{-\frac{\phi}{4}} 2 l_\sigma \, . \tag{56}$$

g_s	$M_s/M_{\rm GUT}$	$M_{\rm open}/M_{\rm GUT}$
10^{-1}	56.2	502.0
10^{-2}	31.6	158.7
10^{-3}	17.8	50.2
10^{-4}	10.0	15.9
10^{-5}	5.6	5.0
10^{-6}	3.2	1.6
10^{-7}	1.8	0.5

Table 1: The string-scale M_s and the mass of open string states $M_{\rm open}$ in units of the GUT scale $M_{\rm GUT} = 2 \times 10^{16}\,{\rm GeV}$ for various perturbative values of the string coupling constant g_s.

An open string stretching between both D3-brane stacks gives rise to a massive state. In the low-energy frame this mass then becomes

$$M_{\rm open} = V_5^{1/8} e^{-\frac{\phi}{4}} 2 l_\sigma T = 2lT = \frac{M_s^2}{2\pi M_{\rm GUT}} \ . \tag{57}$$

For given g_s the string-scale and the mass of the open string states is therefore fixed. We present them in table 1 for various values of g_s. For values of g_s larger than 10^{-6} the open string state masses lie at or above the GUT scale with similar values for the string-scale.

The result that the open string state masses exceed the GUT scale is indeed welcome because we will see that these open string states will also give rise to the grand unified leptoquark X and Y gauge bosons which mediate proton decay. Already at tree-level the proton's lifetime, which is given by the inverse of its decay width Γ_p, turns out to be proportional to the fourth power of the X and Y mass

$$\Gamma_p^{-1} \propto M_{\rm open}^4 \ . \tag{58}$$

A mass for the stretched open string states and therefore the leptoquark gauge bosons larger than the GUT scale will therefore raise the proton's lifetime and could be crucial to avoid conflict of supersymmetric GUT theories with proton decay experiments. More specifically in supergravity SU(5) GUT theories there exist dimension five baryon violating terms in the Lagrangean. They give rise to effective dimension six operators which allow for several decay modes of the proton. However, they are all suppressed by the mass of the color triplet Higgs boson [53]. We will see that the color triplet Higgs boson originates

as well from an open string stretched between both D3-brane stacks so that its mass is also given by $M_{\rm open}$. Detailed studies (for a review see [54]) imply that $M_{\rm open} > M_{\rm GUT}$ by almost a factor of 10 for consistency with present observations on the proton's lifetime. This hierarchy is quite unpleasant in supersymmetric field theory SU(5) GUT models as it requires that some couplings in the superpotential unnaturally have to be much larger than one [54]. On the other hand we readily obtain the required hierarchy between $M_{\rm open}$ and $M_{\rm GUT}$ for not too small values of g_s, ensuring a longer lifetime for the proton.

We also see that the string-scale comes out close to its traditional high value. Low string-scale scenarios in which M_s is lowered to the TeV scale or the intermediate scale 10^{11} GeV require a considerable finetuning of g_s to very small values and would be in conflict with observation since $M_{\rm open}$ would likewise be much smaller in these scenarios leading to rapid proton decay. We will next study the spectrum of possible matter and gauge field states arising from the open strings in the D3-brane picture.

6 Gauge Fields, Higgs Fields and Doublet-Triplett Splitting

For the two D3-brane stacks with a small split of the second stack into $2+1$ D3-branes, we will now examine the open strings, with two orientations each, which are depicted in fig.4 together with their transformation properties under SU(3) and SU(2). In what follows open strings connecting the SU(3) and the SU(2) brane stacks directly won't play a role. They might be projected out by the imposition of an appropriate discrete symmetry. Let us first determine the U(1) brane charges x and y of the two open string states of fig.4. For this imagine in a Gedankenexperiment that all six D3-branes were initially placed on top of each other giving rise to a U(6) gauge group. The Chan-Paton degrees of freedom at the endpoints of an open string attached to this U(6) brane stack transform as $\mathbf{6}$ resp. $\bar{\mathbf{6}}$. Hence the open string state transforms under the adjoint $\mathbf{6} \times \bar{\mathbf{6}}$. By separating the D3-branes into the positions of fig.4 we break the U(6) gauge symmetry into U(3)×U(2)×U(1). To determine the U(1) charges x and y we have to look for states in the product $\mathbf{6} \times \bar{\mathbf{6}}$ which transform as $(\mathbf{3}, \mathbf{1})$ resp. $(\mathbf{1}, \mathbf{2})$ under the SU(3)×SU(2) part of the first two factors and read off their U(1) charge (the two U(1) factors contained in U(3) and U(2) correspond to the center-of-mass motion of the corresponding brane stacks. When coupled to gravity these decouple from the low-energy spectrum as the corresponding photons acquire mass at the string scale. The remaining low-energy gauge

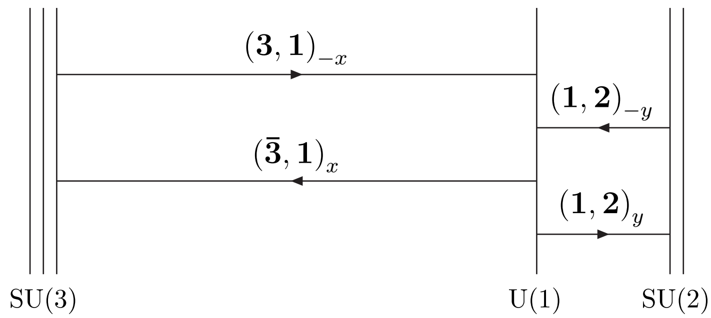

Figure 4: The relevant open strings of the D3-brane setup. The state $(\mathbf{n},\mathbf{m})_z$ transforms as $\mathbf{n}$ under SU(3), as $\mathbf{m}$ under SU(2) and bears U(1) charge z. For better visibility we magnify the small split into $2+1$ branes of the second D3-brane stack. Note that the fundamental $\mathbf{2}$ of SU(2) is pseudoreal.

group SU(3)×SU(2)×U(1) can be thought of as arising from a broken SU(5) GUT which we will discuss below). This yields

$$x = 2\ , \qquad y = 3\ . \tag{59}$$

We have thus two open string states

$$\zeta_u = (\mathbf{3},\mathbf{1})_{-2}\ ,\ \bar{\zeta}_d = (\bar{\mathbf{3}},\mathbf{1})_{2} \tag{60}$$

with mass at or above the GUT scale and two states

$$H_u = (\mathbf{1},\mathbf{2})_{3}\ ,\ \bar{H}_d = (\mathbf{1},\mathbf{2})_{-3} \tag{61}$$

with almost vanishing mass, say at the TeV scale or less, controlled by the little split between the U(1) and the SU(2) branes (see fig.5).

Depending on whether we consider open strings in the four non-compact directions with Neumann boundary conditions along the D3-branes or in the six compact directions with Dirichlet boundary conditions transverse to the D3-branes, these give rise to either gauge bosons (vector superfields) or Higgs scalars (chiral superfields). We have denoted by $\zeta_u, \bar{\zeta}_d$ the color triplet Higgs bosons and by $H_u, \bar{H}_d$ the weak doublet Higgs bosons and will see later that they fit into multiplets of a supersymmetric SU(5) GUT theory with gauge symmetry broken spontaneously to SU(3)×SU(2)×U(1). As a consequence

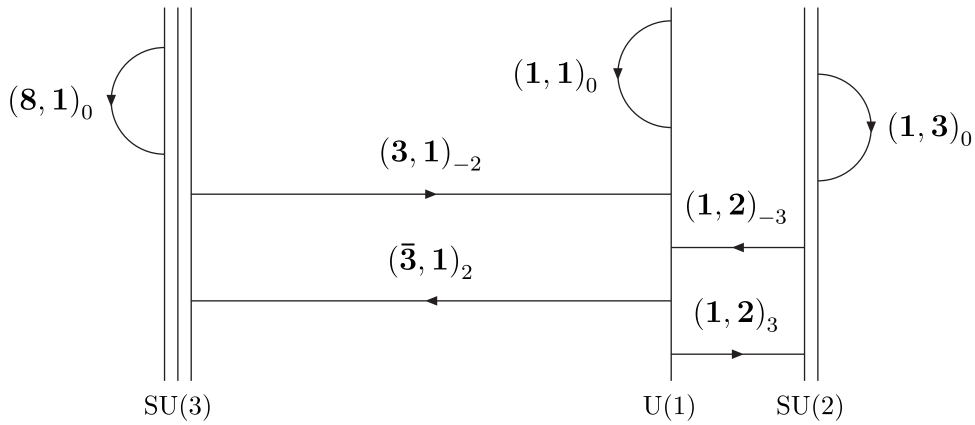

Figure 5: Open strings and their SU(3)×SU(2)×U(1) transformations. Those which start and end on the same D3-brane stack will give rise to SM gauge fields and part of the Higgs bosons filling the **24** adjoint Higgs multiplet of a supersymmetric SU(5) GUT. Those open strings which stretch between different D3-branes will provide the **5** and $\bar{\mathbf{5}}$ SU(5) Higgs multiplets. Gauge fields originate from open strings vibrating in the non-compact directions with Neumann boundary conditions at their ends while Higgs fields originate from open strings vibrating in the internal directions satisfying Dirichlet boundary conditions on the D3-branes. The geometry directly implies a mass hierarchy between color triplet and weak doublet Higgs fields.

of the large separation between the SU(3) D3-branes and the U(1) D3-brane and on the other hand the short separation between the SU(2) D3-branes and the U(1) D3-brane, a *mass-hierarchy between color triplets states and weak doublet states* follows directly from the geometry. We will come back to this hierarchy later when we identify these states with SU(5) Higgs fields and come to the doublet-triplet splitting problem.

In addition, we have also open strings which start and end on the same stack of D3-branes. In the NS sector these give rise to 4-dimensional gauge-fields $A^{ij,\mu} = b^{\mu}_{-1/2}|k_4; i, j\rangle$ (i, j representing the Chan-Paton labels) in the non-compact directions with Neumann boundary conditions at the open string endpoints. The momenta k_4 are along the four non-compact directions filled by the D3-branes. Altogether this leads to three massless gauge-bosons

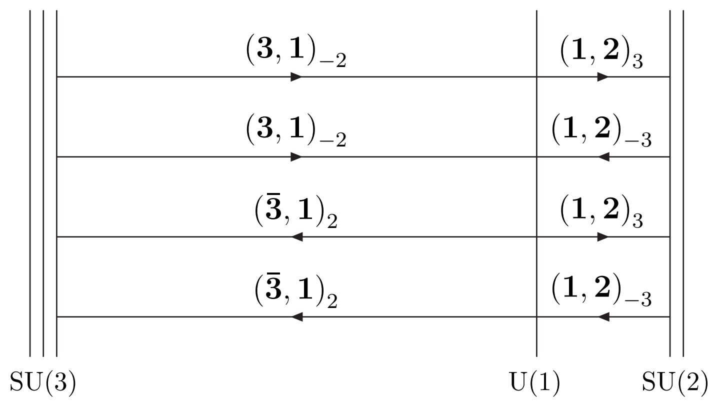

Figure 6: States which arise from the composition of two stretched open strings.

$$B = (\mathbf{1},\mathbf{1})_0 \ , \ W^i = (\mathbf{1},\mathbf{3})_0 \ , \ A^\alpha = (\mathbf{8},\mathbf{1})_0 \tag{62}$$

which can be identified with the SM gauge bosons (see fig.5). In addition in the compact directions transverse to the D3-branes the open strings satisfy Dirichlet boundary conditions and give rise to scalar fields $\phi^{ij,m} = b^m_{-1/2}|k_4; i, j\rangle$; $m = 4, \ldots, 9$, parameterizing the positions of the D3-branes. These states transform in the same way as the gauge fields under SU(3)×SU(2)×U(1)

$$\sigma_1 = (\mathbf{1},\mathbf{1})_0 \ , \ \sigma_2 = (\mathbf{1},\mathbf{3})_0 \ , \ \sigma_3 = (\mathbf{8},\mathbf{1})_0 \tag{63}$$

and will later build part of the SU(5) adjoint **24** Higgs multiplet.

Finally we can generate four more states through the composition of two stretched open strings. These composite states, which are marginally stable due to supersymmetry, will also have large masses M_{open} at or above the GUT scale. The four different possibilities depicted in fig.6 give us the states (similar composites appeared also recently in [55], [56])

$$(\mathbf{3},\mathbf{1})_{-2} \otimes (\mathbf{1},\mathbf{2})_3 \ \ \rightarrow (\mathbf{3},\mathbf{2})_1 \tag{64}$$
$$(\mathbf{3},\mathbf{1})_{-2} \otimes (\mathbf{1},\mathbf{2})_{-3} \rightarrow (\mathbf{3},\mathbf{2})_{-5} \tag{65}$$
$$(\bar{\mathbf{3}},\mathbf{1})_{2} \otimes (\mathbf{1},\mathbf{2})_3 \ \ \rightarrow (\bar{\mathbf{3}},\mathbf{2})_5 \tag{66}$$
$$(\bar{\mathbf{3}},\mathbf{1})_{2} \otimes (\mathbf{1},\mathbf{2})_{-3} \rightarrow (\bar{\mathbf{3}},\mathbf{2})_{-1} \ . \tag{67}$$

Again, depending on whether we consider the strings in the non-compact or the compact directions we will get gauge bosons or Higgs fields with the indicated SU(3)×SU(2)×U(1) transformation properties. Two of them

$$X = (\mathbf{3}, \mathbf{2})_{-5} \; , \; \bar{Y} = (\bar{\mathbf{3}}, \mathbf{2})_5 \tag{68}$$

will account for the twelve heavy X and $\bar{Y}$ leptoquark gauge bosons of SU(5) and the remaining Higgs bosons

$$\sigma_4 = (\mathbf{3}, \mathbf{2})_{-5} \; , \; \sigma_5 = (\bar{\mathbf{3}}, \mathbf{2})_5 \tag{69}$$

needed to fill the adjoint **24** Higgs multiplet of SU(5).

7 Light MSSM Matter Fields

So far we have seen how gauge and Higgs fields can arise from open strings in the D3-brane setup. But we still need to incorporate light matter fields (with mass at or below the TeV scale) which could account for the matter content of the minimal supersymmetric Standard Model (MSSM). The mechanism by which these states can emerge along with the heavy GUT states is rather simple. Suppose that the compactification manifold K_6 has non-trivial fundamental group $\pi_1(K_6) \neq 0$. This is the case e.g. for orbifold compactifications which are based on torus compactifications. Already the n-torus T^n has a non-trivial fundamental group $\pi_1(T^n) = \mathbf{Z} \oplus \ldots \oplus \mathbf{Z}$ with n summands $\mathbf{Z}$. The geometry will then look like in fig.7 and we can orient the direction X^5 around the non-simply connected path. While the open strings which we have been discussing so far stretch around X^5 and therefore have to wind around the full loop to connect both stacks of D3-branes, we can also have very short open strings which connect the two stacks via the orthogonal dimensions $X^6, \ldots, X^9$. Depending on the distance between the D3-brane stacks in the directions $X^6, \ldots, X^9$ the mass of these "short" open strings can be made very small. This represents a simple way to obtain both heavy GUT excitations and light matter fields from open strings stretching between the same stacks of D3-branes.

The transformation of the "short" open string states under SU(3)×SU(2)×U(1) follows the same lines as discussed before. In particular we can identify the MSSM lepton and quark chiral superfields $L, \bar{D}, Q, \bar{U}, \bar{E}$ with the open strings shown in fig.8. L and $\bar{D}$ arise directly from two open strings

$$L = (\mathbf{1}, \mathbf{2})_{-3} \; , \; \bar{D} = (\bar{\mathbf{3}}, \mathbf{1})_2 \tag{70}$$

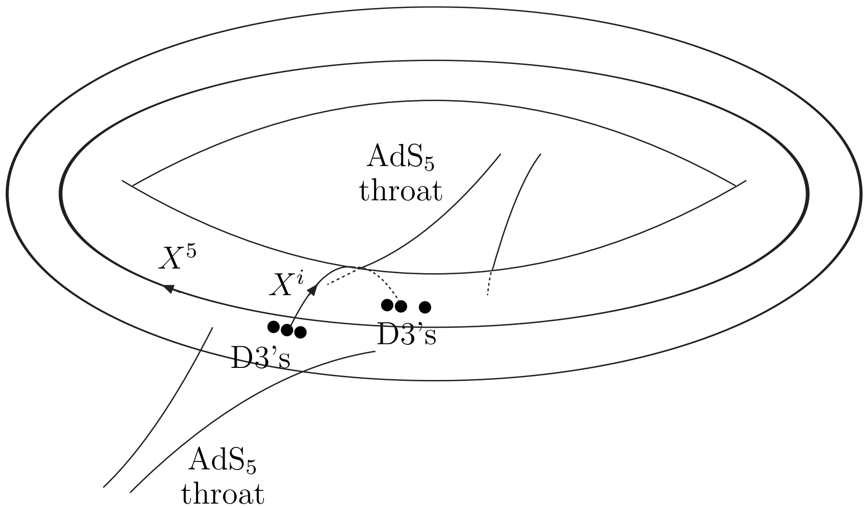

Figure 7: On non-simply connected compactification spaces light MSSM matter fields can arise from open strings connecting D3-branes via the short paths along directions X^i, while heavy GUT fields are generated from open strings connecting the same D3-branes via the long path around X^5. The D3-branes appear as points on the compactification space.

while $Q, \bar{U}, \bar{E}$ arise from their compositions. More specifically the $Q, \bar{U}, \bar{E}$ chiral superfields arise from the following compositions

$$Q = (\mathbf{3}, \mathbf{2})_1 = (\mathbf{3}, \mathbf{1})_{-2} \otimes (\mathbf{1}, \mathbf{2})_3 \tag{71}$$

$$\bar{U} = (\bar{\mathbf{3}}, \mathbf{1})_{-4} \subset (\mathbf{3}, \mathbf{1})_{-2} \otimes (\mathbf{3}, \mathbf{1})_{-2} \tag{72}$$

$$\bar{E} = (\mathbf{1}, \mathbf{1})_6 \subset (\mathbf{1}, \mathbf{2})_3 \otimes (\mathbf{1}, \mathbf{2})_3 \ , \tag{73}$$

whose geometrical meaning is given in fig.8. For this we have used $\mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} + \mathbf{6}$ and $\mathbf{2} \otimes \mathbf{2} = \mathbf{1} + \mathbf{3}$ in the last two cases and picked the antisymmetric part while dismissing the symmetric one.

This completes our discussion of the generic features of the original 5-dimensional 3-brane configuration when lifted to IIB string-theory with D3-branes. We have seen that heavy and light gauge and Higgs fields arise with a mass hierarchy dictated by the geometry. Also light MSSM matter chiral superfields with the correct SU(3)×SU(2)×U(1) transformation can be accommodated. We will next show how these states in fact combine into multiplets of a supersymmetric GUT theory with gauge group SU(5). Of course the

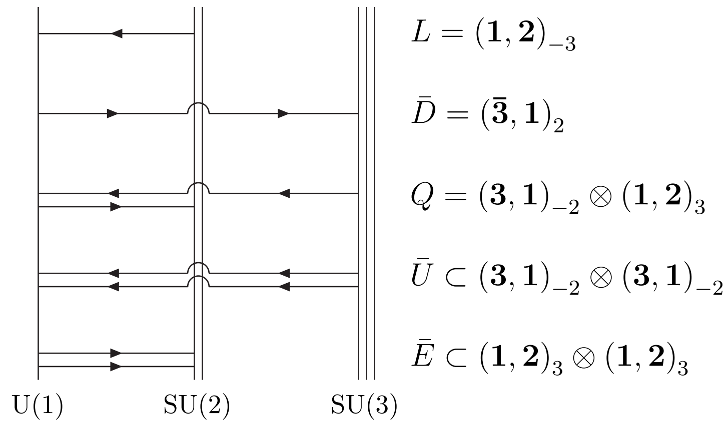

Figure 8: Light MSSM quark and lepton chiral superfields $L, \bar{D}, Q, \bar{U}, \bar{E}$ arise from open string states $(L, \bar{D})$ and compositions thereof $(Q, \bar{U}, \bar{E})$. The open strings connect the D3-branes via the short paths along directions $X^i \neq X^5$.

states which we have discussed constitute a subset of all in principle possible string-states. It will be left to a case by case model analysis to apply suitable discrete projections to get rid of unwanted additional states. This is, however, beyond the scope of the present paper whose concern is with general properties. There is also the important issue of breaking the initial $\mathcal{N} = 4$ supersymmetry preserved by the D3-brane stacks to $\mathcal{N} = 1$. This will be discussed in the next but one section.

8 SU(5) Grand Unification

Let us now show that the string states described so far actually fill out the multiplets of a supersymmetric SU(5) grand unified theory. The gauge group is, however, spontaneously broken to SU(3)×SU(2)×U(1). This breaking is spontaneous because the masses are generated in the string-theory description by separating D3-branes from each other, a mechanism which is known to correspond to a Higgs mechanism in the effective field theory description.

It is clear that when all six D3-branes would lie on top of each other an unbroken SU(6) gauge group would be restored. One therefore expects that when the branes are separated into three SU(3) branes, two SU(2) branes and one U(1) brane, as in our case,

that one should be able to recover the spectrum of a supersymmetric GUT with gauge group SU(6), or a subgroup thereof, broken spontaneously to the SM gauge group. This identification can indeed be carried out for SU(6) but is complicated by the fact that the Higgs and matter content required for a supersymmetric SU(6) GUT is quite involved and includes several SU(3)×SU(2)×U(1) singlets [57]. We will therefore focus on the supersymmetric SU(5) GUT theory which has in its minimal formulation a quite succinct field content.

The minimal field content for a supersymmetric SU(5) GUT requires, next to a vector superfield transforming as the adjoint $V^a = \mathbf{24}$, chiral superfields for three generations of chiral fermions transforming as the $\psi^f = \bar{\mathbf{5}}$ and $\chi^f = \mathbf{10}$ ($f = 1, 2, 3$ labeling the families). In addition there is the adjoint $\Sigma^a = \mathbf{24}$ of scalars necessary to break the SU(5) symmetry to the SM gauge group and two further fundamental and anti-fundamental Higgs scalars $H = \mathbf{5}$, $\bar{H} = \bar{\mathbf{5}}$. These are associated with the electroweak symmetry breaking. The SU(5) field content can therefore be summarized as

- Gauge: $V^a = \mathbf{24}$
- Matter: $\chi^f = \mathbf{10}$, $\psi^f = \bar{\mathbf{5}}$
- Higgs: $\Sigma^a = \mathbf{24}$, $H = \mathbf{5}$, $\bar{H} = \bar{\mathbf{5}}$

Broken down to the SM gauge group, we have the following decompositions for the fundamental $\mathbf{5}$, the antisymmetric tensor $\mathbf{10}$ and adjoint $\mathbf{24}$ under $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$

$$\begin{aligned}
\mathbf{24} &= (\mathbf{1},\mathbf{1})_0 + (\mathbf{1},\mathbf{3})_0 + (\mathbf{8},\mathbf{1})_0 + (\mathbf{3},\mathbf{2})_{-5} + (\bar{\mathbf{3}},\mathbf{2})_5 \\
\mathbf{10} &= (\mathbf{1},\mathbf{1})_6 + (\bar{\mathbf{3}},\mathbf{1})_{-4} + (\mathbf{3},\mathbf{2})_1 \\
\mathbf{5} &= (\mathbf{1},\mathbf{2})_3 + (\mathbf{3},\mathbf{1})_{-2} \ .
\end{aligned}$$

The SU(5) superfields hence break up into the following $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ superfields

- Gauge: $V^a = \mathbf{24} \ \rightarrow$ massless: $B = (\mathbf{1},\mathbf{1})_0$, $W^i = (\mathbf{1},\mathbf{3})_0$, $A^\alpha = (\mathbf{8},\mathbf{1})_0$
  massive: $X = (\mathbf{3},\mathbf{2})_{-5}$, $\bar{Y} = (\bar{\mathbf{3}},\mathbf{2})_5$
- Matter: $\chi^f = \mathbf{10} \ \rightarrow$ light: $\bar{E} = (\mathbf{1},\mathbf{1})_6$, $\bar{U} = (\bar{\mathbf{3}},\mathbf{1})_{-4}$, $Q = (\mathbf{3},\mathbf{2})_1$
  $\psi^f = \bar{\mathbf{5}} \ \rightarrow$ light: $L = (\mathbf{1},\mathbf{2})_{-3}$, $\bar{D} = (\bar{\mathbf{3}},\mathbf{1})_2$

- Higgs: $\Sigma^a = \mathbf{24} \;\rightarrow\;$ massive: $\sigma_1 = (\mathbf{1},\mathbf{1})_0$, $\sigma_2 = (\mathbf{1},\mathbf{3})_0$, $\sigma_3 = (\mathbf{8},\mathbf{1})_0$, $\sigma_4 = (\mathbf{3},\mathbf{2})_{-5}$, $\sigma_5 = (\bar{\mathbf{3}},\mathbf{2})_5$

  $H = \mathbf{5} \;\rightarrow\;$ light: $H_u = (\mathbf{1},\mathbf{2})_3$
  massive: $\zeta_u = (\mathbf{3},\mathbf{1})_{-2}$

  $\bar{H} = \bar{\mathbf{5}} \;\rightarrow\;$ light: $\bar{H}_d = (\mathbf{1},\mathbf{2})_{-3}$
  massive: $\bar{\zeta}_d = (\bar{\mathbf{3}},\mathbf{1})_2$

where "massive" means masses at or above the GUT scale while "light" indicates hierarchically smaller masses at or below the TeV scale. B, W^i, A^α denote the vector superfields containing the respective SM gauge bosons, while X and $\bar{Y}$ stand for the leptoquark gauge bosons. Quarks/squarks are contained in $Q, \bar{U}, \bar{D}$ and leptons/sleptons in $L, \bar{E}$. Finally, the heavy Higgs/higgsinos come as color triplets $\zeta_u, \bar{\zeta}_d$ while the light Higgs/higgsinos transform as weak doublets, denoted by $H_u, \bar{H}_d$.

As we saw in detail earlier, these were precisely the states which were generated by the open strings in the D3-brane setup. Moreover, the states required to be "massive" came out indeed with a large mass $M_{\rm open}$ at or greater than the GUT scale, the states labeled as "light" came out with small masses and the states labeled as "massless" came out to be massless. The only exception to this correct assignment of masses in the string description are the Higgs bosons $\sigma_1, \sigma_2, \sigma_3$ which appeared to be massless. It is at this point where moduli stabilization would set in. Namely, the fields $\sigma_1, \sigma_2, \sigma_3$, coming from the zero modes of the open string components along the compact directions, represent the moduli which describe the positions of the D3-branes. Once these positions get stabilized, a potential will fix the values of $\sigma_1, \sigma_2, \sigma_3$ and render them massive. These masses will naturally take values around the string-scale which is also at or above the GUT scale (see table 1). We also see that the U(1) can indeed be identified with the SM $\mathrm{U}(1)_Y$ hypercharge, and therefore remains anomaly-free, as the U(1) charges of all fields exactly match those of the SM $\mathrm{U}(1)_Y$. Moreover, with the identification of the color triplet and weak doublet states as Higgs chiral superfields, we obtain that the 3-brane separation $2l = M_{\rm GUT}^{-1}$, required for a sufficient suppression of the 4-dimensional vacuum energy, implies a simple resolution of the doublet-triplet splitting problem in the string-theory description of the SU(5) GUT theory. The number of families will however be model dependent and follows from topological data of the compactification manifold such as the Euler character.

9 Supersymmetry Breaking

The $\mathcal{N}=1$ vector and chiral superfields which made up the gauge bosons, matter fermions and Higgs fields of the spontaneously broken SU(5) supersymmetric GUT resp. MSSM, came from open strings attached to parallel D3-branes. They are thus originating from $\mathcal{N}=4$ vector supermultiplets. One $\mathcal{N}=4$ vector supermultiplet contains three $\mathcal{N}=1$ chiral superfields plus one $\mathcal{N}=1$ vector supermultiplet. Let us finally briefly address how one might break the $\mathcal{N}=4$ supersymmetry to an $\mathcal{N}=1$ supersymmetry.

The fermions originate from the Ramond-sector of the open strings and in uncompactified ten dimensions would be described by a Majorana-Weyl spinor $u_\alpha^{ij}|\alpha;k_{10};i,j\rangle$. Here, $\alpha=1,\dots,8$ is a spinor-index running over the physical on-shell degrees of freedom, i,j are the Chan-Paton labels and k_{10} is the 10-dimensional momentum. By dimensional reduction to four dimensions, u_α turns into four two-component Weyl-spinors λ_a^{ij}; $a=1,\dots,4$. In a 4-dimensional $\mathcal{N}=1$ description λ_4^{ij} gets combined with the gauge-field A_μ^{ij} of the Neveu-Schwarz sector into an $\mathcal{N}=1$ vector-superfield, while the remaining three spinors $\lambda_1^{ij},\lambda_2^{ij},\lambda_3^{ij}$ are each paired with two real Neveu-Schwarz scalars $(\phi_4^{ij},\phi_7^{ij}),(\phi_5^{ij},\phi_8^{ij}),(\phi_6^{ij},\phi_9^{ij})$ to build the three chiral superfields

$$Z_a^{ij}=(\phi_{a+3}^{ij}+i\phi_{a+6}^{ij},\lambda_a^{ij})\ ,\quad a=1,2,3\ . \tag{74}$$

Together the three chiral superfields plus the single vector superfield make up the $\mathcal{N}=4$ vector supermultiplet. Hence we naturally arrive at a multiplicity of three for the chiral matter fermions. It would be interesting to explore in concrete models the connection between this multiplicity and the number three of fermion families which we will leave to future work.

One way of breaking the $\mathcal{N}=4$ supersymmetry to an $\mathcal{N}=1$ supersymmetry is, at the field-theory level, by adding masses to the three chiral superfields in an $\mathcal{N}=1$ supersymmetry preserving way [58], [59]. For this one supplements the $\mathcal{N}=1$ superpotential by the mass terms

$$\Delta W\sim\sum_{a=1}^{3}m_a\mathrm{tr}Z_a^2\ . \tag{75}$$

This lifts the mass degeneracy of the chiral superfields by construction. What makes these mass perturbations not ad hoc and even natural in string-theory, is that the fact that they correspond to magnetic 3-form fluxes H (see [59] for details). As we have seen earlier such fluxes are required in generic string-theory compactifications by the tadpole cancellation condition.

Let us finally point out yet another way to break, with the help of additional D7-branes, $\mathcal{N} = 4$ to $\mathcal{N} = 1$ supersymmetry. The generic tangent space group of a 6-dimensional compactification manifold is SO(6). The D3-branes were all transverse to the compactification manifold and will therefore not influence its tangent space group. This will change, however, when additional D7-branes are present in the type IIB string-theory vacuum. Consider for instance three D7-branes with worldvolume along the directions 01234578, 01234679 and 01235689. Together, they preserve 1/8 of the initial 32 IIB supercharges, and therefore leave us with the desired $\mathcal{N} = 1$ supersymmetry in four dimensions. Moreover, the supersymmetry preserved by the D7-branes is compatible with the one preserved by the D3-branes. To see this, let us write down the conditions imposed on the supersymmetry parameters ϵ_L, ϵ_R (16-component Majorana-Weyl spinors in IIB of the same chirality) by the presence of the D7-branes. These are

$$\epsilon_L = \Gamma_{D3}\Gamma_{R_1}\epsilon_R \; ; \quad \Gamma_{R_1} = \Gamma^4\Gamma^5\Gamma^7\Gamma^8 \; , \;\; \Gamma_{D3} = \Gamma^0\Gamma^1\Gamma^2\Gamma^3 \tag{76}$$

$$\epsilon_L = \Gamma_{D3}\Gamma_{R_2}\epsilon_R \; ; \quad \Gamma_{R_2} = \Gamma^4\Gamma^6\Gamma^7\Gamma^9 \tag{77}$$

$$\epsilon_L = \Gamma_{D3}\Gamma_{R_3}\epsilon_R \; ; \quad \Gamma_{R_3} = \Gamma^5\Gamma^6\Gamma^8\Gamma^9 \; . \tag{78}$$

Taken together, they imply that

$$\epsilon_L = \Gamma_{D3}\epsilon_R \; , \tag{79}$$

which is exactly the condition imposed by a 4-dimensional spacetime filling D3-brane. Therefore the D3-branes preserve the $\mathcal{N} = 1$ supersymmetry preserved by the D7-branes and can be placed at the common 4-dimensional intersection of the three intersecting D7-branes. The presence of the D7-branes will break the SO(6) tangent space group down to

$$\mathrm{SO}(6) \supset \mathrm{SO}(2) \times \mathrm{SO}(2) \times \mathrm{SO}(2) = \mathrm{U}(1) \times \mathrm{U}(1) \times \mathrm{U}(1) \; . \tag{80}$$

This entails a corresponding split of the three chiral superfields, initially combined into the $\mathcal{N} = 4$ vector multiplet, into three separate multiplets as each one transforms under a separate U(1). Thus the degeneracy between them gets lifted and they can acquire different $\mathcal{N} = 1$ masses.

Acknowledgements

We would like to thank Dieter Lüst, Lisa Randall and Raman Sundrum for discussion and correspondence.

A Warped Geometry and Effective D=4 Action for Unequal Wall Tensions

In this appendix we analyze the warped geometry and effective action for unequal 3-brane tensions $T_1 \neq T_2$. Let us emphasize that in string-theory all D3-branes come with the same tension, so that equal tensions simply required an equal number of D3-branes. The unequal tension case can nevertheless be relevant in the effective 5-dimensional description. In this case the Ansatz (3) yields the solution

$$A(x^5) = \frac{k_1}{2}\left|x^5 + l\right| + \frac{k_2}{2}\left|x^5 - l\right| = \begin{cases} \frac{1}{2}K_{12}x^5 + \frac{1}{2}k_{12}l & , \ x^5 \geq l \\ \frac{1}{2}k_{12}x^5 + \frac{1}{2}K_{12}l & , \ -l \leq x^5 \leq l \\ -\frac{1}{2}K_{12}x^5 - \frac{1}{2}k_{12}l & , \ x^5 \leq -l \end{cases} , \tag{81}$$

with constants $K_{12} = k_1 + k_2$ and $k_{12} = k_1 - k_2$. Without loss of generality, we can assume that $k_1 \geq k_2$ subsequently. The function $A(x^5)$ which determines the warp-factor is displayed in fig.9. The corresponding warp-factor $e^{-A(x^5)}$ is bounded from above by $e^{-k_2 l}$ over the whole fifth dimension. Again we have set an arbitrary integration constant which could be added to $A(x^5)$ to zero. The Einstein equations (4) determine the stepwise constant bulk cosmological constant

$$\Lambda(x^5) = \begin{cases} \Lambda_e \ , & |x^5| \geq l \\ \Lambda_i \ , & |x^5| < l \end{cases} = -\frac{3M_5^3}{4}\begin{cases} K_{12}^2 \ , & |x^5| \geq l \\ k_{12}^2 \ , & |x^5| < l \end{cases} \tag{82}$$

and 3-brane tensions

$$T_1 = 3M_5^3 k_1 \ , \qquad T_2 = 3M_5^3 k_2 \ . \tag{83}$$

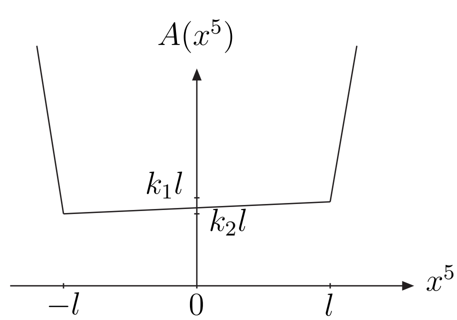

Figure 9: The function $A(x^5)$ which determines the warp-factor.

The next task is again the determination of the effective 4-dimensional action by integration over the fifth dimension. Along the same lines as before, by employing (12), we get for the Einstein-Hilbert term

$$
\begin{aligned}
S_{EH} &= -\int d^4x\sqrt{g}M_5^3\left(R(g)\int_{-\infty}^{\infty}dx^5e^{-A}+\int_{-\infty}^{\infty}dx^5e^{-2A}\left[5(A')^2-4A''\right]\right)\\
&= -e^{-K_{12}l/2}\int d^4x\sqrt{g}M_5^3\Bigg(4R(g)\Big[\frac{1}{K_{12}}\cosh\left(\frac{k_{12}l}{2}\right)+\frac{1}{k_{12}}\sinh\left(\frac{k_{12}l}{2}\right)\Big]\\
&\quad+\frac{5}{4}e^{-K_{12}l/2}\Big[2K_{12}\cosh\left(k_{12}l\right)+2k_{12}\sinh\left(k_{12}l\right)\Big]\\
&\quad-4e^{-K_{12}l/2}\Big[k_1e^{k_{12}l}+k_2e^{-k_{12}l}\Big]\Bigg)\ . \qquad (84)
\end{aligned}
$$

For the brane-terms and bulk cosmological constant term we get

$$
\begin{aligned}
S_{SM_1}+S_{SM_2}+S_\Lambda = &-e^{-K_{12}l}\int d^4x\sqrt{g}\Bigg(e^{k_{12}l}T_1+e^{-k_{12}l}T_2\\
&+2\frac{\Lambda_e}{K_{12}}\cosh(k_{12}l)+2\frac{\Lambda_i}{k_{12}}\sinh(k_{12}l)\Bigg)\ . \qquad (85)
\end{aligned}
$$

Pulling out an overall constant factor of $e^{-K_{12}l/2}$ in front, the final effective action reads

$$
\begin{aligned}
&S_{EH}+S_{SM_1}+S_{SM_2}+S_\Lambda\\
&= -e^{-K_{12}l/2}\int d^4x\sqrt{g}\Bigg(4M_5^3R(g)\Big[\frac{1}{K_{12}}\cosh\left(\frac{k_{12}l}{2}\right)+\frac{1}{k_{12}}\sinh\left(\frac{k_{12}l}{2}\right)\Big]\\
&\quad+\frac{5}{2}M_5^3e^{-K_{12}l/2}\Big[K_{12}\cosh\left(k_{12}l\right)+k_{12}\sinh\left(k_{12}l\right)\Big]+e^{-K_{12}l/2}\Big[e^{k_{12}l}(T_1-4k_1M_5^3)\\
&\quad+e^{-k_{12}l}(T_2-4k_2M_5^3)+2\frac{\Lambda_e}{K_{12}}\cosh(k_{12}l)+2\frac{\Lambda_i}{k_{12}}\sinh(k_{12}l)\Big]\Bigg)\ . \qquad (86)
\end{aligned}
$$

By the same reasoning as explained in the main text, we drop the overall constant factor and arrive at the effective action

$$
S_{D=4} = -\int d^4x\sqrt{g}\left(M_4^2R(g)+\Lambda_4\right)\ , \qquad (87)
$$

with effective 4-dimensional Planck-scale M_4 and 4-dimensional vacuum energy Λ_4 given by

$$
M_4^2 = 4M_5^3\Big[\frac{1}{K_{12}}\cosh\left(\frac{k_{12}l}{2}\right)+\frac{1}{k_{12}}\sinh\left(\frac{k_{12}l}{2}\right)\Big] \qquad (88)
$$

$$
\begin{aligned}
\Lambda_4 = e^{-K_{12}l/2}\Bigg(&\frac{5}{2}M_5^3\Big[K_{12}\cosh\left(k_{12}l\right)+k_{12}\sinh\left(k_{12}l\right)\Big]+\Big[e^{k_{12}l}(T_1-4k_1M_5^3)\\
&+e^{-k_{12}l}(T_2-4k_2M_5^3)+2\frac{\Lambda_e}{K_{12}}\cosh(k_{12}l)+2\frac{\Lambda_i}{k_{12}}\sinh(k_{12}l)\Big]\Bigg)\ . \qquad (89)
\end{aligned}
$$

Again, there exists an exponential suppression-factor $e^{-K_{12}l/2}$ multiplying the vacuum energy, which allows to suppress Λ_4 down to its observational value for generic values $k_1, k_2 \approx M_{Pl}$. When the values (82),(83) for $T_1, T_2, \Lambda_e, \Lambda_i$ are substituted into the obtained effective action, we arrive at a vanishing Λ_4, which checks the derivation of the action, since in this case the fine-tuning of the parameters should guarantee a flat 4-dimensional metric $g_{\mu\nu} = \eta_{\mu\nu}$. For the particular case of coinciding brane-tensions, $T_1 = T_2 = T$ (which entails $k_1 = k_2 = k$), we arrive at the effective action given by (28),(29),(30), which was discussed in the main text.

Again, let us now relax the fine-tuning

$$\Lambda(x^5) = \left\{ \begin{array}{ll} \Lambda_e \ , & |x^5| \geq l \\ \Lambda_i \ , & |x^5| < l \end{array} \right. = -\frac{1}{12M_5^3} \left\{ \begin{array}{ll} (T_1+T_2)^2 \ , & |x^5| \geq l \\ (T_1-T_2)^2 \ , & |x^5| < l \end{array} \right. , \tag{90}$$

between the bulk cosmological constant and the 3-brane tensions which implies a non-flat 4-dimensional metric $g_{\mu\nu} \neq \eta_{\mu\nu}$ in the Ansatz

$$ds^2 = e^{-A(x^5)} g_{\mu\nu} dx^\mu dx^\nu + (dx^5)^2 \ . \tag{91}$$

Depending on whether Λ_4 is positive or negative, the metric $g_{\mu\nu}$ describes either a de Sitter or anti-De Sitter spacetime. Our interest lies in the de Sitter case. From (89) it is evident, that in order to arrive at an exponentially small Λ_4, the difference between both 3-brane tensions cannot be too large but has to be constrained by

$$k_1 - k_2 \equiv k_{12} \lesssim \frac{1}{l} = 2M_{\mathrm{GUT}} \ . \tag{92}$$

Most naturally we would expect values such as $k_1, k_2 \approx M_{Pl}$, $T_1, T_2 \approx M_{Pl}^4$, $\Lambda_e \approx M_{Pl}^5$ and for the 5-dimensional Planck-scale $M_5 \approx M_{Pl}$. If also Λ_i does not exceed $(3 \times 10^{18}\mathrm{GeV})^5$, which is a bit larger than the reduced Planck mass, then we recognize from (89) that the suppression by the exponential pre-factor is just sufficient, in view of (33), to decrease the Planck-valued contributions to the 4-dimensional vacuum energy down to its observed value.

B The Effective Potential for Bulk Scalars in the Case of Unequal Wall-Tensions

For completeness, let us also derive the bulk scalar contribution to the effective potential Λ_4 in the case with unequal brane tensions. With the same action for the bulk scalar

Φ with mass m as in the main text, we obtain for unequal brane tensions the following solution to the field equation

$$\Phi(x^5) = \begin{cases} ae^{(1+\Gamma)A(x^5)} + be^{(1-\Gamma)A(x^5)} & , \ x^5 < -l \\ ce^{(1+\gamma)A(x^5)} + de^{(1-\gamma)A(x^5)} & , \ |x^5| \leq l \\ ee^{(1+\Gamma)A(x^5)} + fe^{(1-\Gamma)A(x^5)} & , \ x^5 > l \end{cases} , \tag{93}$$

where now

$$\Gamma = \sqrt{1 + 4m^2/K_{12}^2} \,, \qquad \gamma = \sqrt{1 + 4m^2/k_{12}^2} \,. \tag{94}$$

In order to obtain a normalizable solution for Φ, we set the coefficients $a = e = 0$. Moreover, imposing continuity of Φ at the position of the 3-branes determines b and f in terms of c and d

$$b = e^{\Gamma k_2 l}\tilde{b} \,, \qquad \tilde{b} = ce^{\gamma k_2 l} + de^{-\gamma k_2 l} \tag{95}$$

$$f = e^{\Gamma k_1 l}\tilde{f} \,, \qquad \tilde{f} = ce^{\gamma k_1 l} + de^{-\gamma k_1 l} \,. \tag{96}$$

To fix the remaining coefficients c and d one would have to plug the above bulk solution into the field equation and integrate out the fifth dimension to incorporate the brane boundary conditions. Since this leads to a complicated cubic equation in the unknowns c and d, it is again easier to determine them by inserting the bulk solution into the scalar action and integrating it over x^5 to arrive at an effective potential for the inter-brane distance $2l$. For positive couplings λ_1, λ_2 this effective potential will be positive definite. Hence, to minimize the potential, we are led to set $\Phi(-l) = v_1$ and $\Phi(l) = v_2$. This allows for a determination of c and d in terms of the vacuum expectation values v_1, v_2

$$c = \frac{v_2 e^{-(1-\gamma)k_1 l} - v_1 e^{-(1-\gamma)k_2 l}}{e^{2\gamma k_1 l} - e^{2\gamma k_2 l}} \,, \qquad d = \frac{v_2 e^{-(1+\gamma)k_1 l} - v_1 e^{-(1+\gamma)k_2 l}}{e^{-2\gamma k_1 l} - e^{-2\gamma k_2 l}} \,. \tag{97}$$

The effective potential[10] then eventually becomes

$$\begin{aligned} V_\Phi(l) = & \frac{k_{12}}{2}\sinh(\gamma k_{12} l)\Big(c^2(\gamma+1)e^{\gamma K_{12} l} + d^2(\gamma-1)e^{-\gamma K_{12} l}\Big) \\ & + \frac{(\Gamma-1)K_{12}}{4}\big(\tilde{b}^2 + \tilde{f}^2\big) \,. \end{aligned} \tag{98}$$

A numerical investigation of this potential shows that, also in the case with differing tensions, a bulk scalar leads generically to an effective potential, which is likewise sufficiently exponentially suppressed.

[10] We use the relations $(1 \pm \gamma)^2 \frac{k_{12}^2}{4} + m^2 = \gamma(\gamma \pm 1)\frac{k_{12}^2}{2}$ and $(1-\gamma^2)\frac{k_{12}^2}{4} + m^2 = 0$.

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